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SPECIFICATION ANALYSIS OF STRUCTURAL CREDIT RISK MODEL WITH STOCHASTIC VOLATILITY

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Abstract
In this research, we conduct specification analysis of structural credit risk models with stochastic volatility using term structure of Credit Default Swap (CDS) spreads and equity volatility form high-frequency return data. We also test five representatives of structural credit risk models using samples of 93 single name CDS contracts from January 2010 - 2022. The model we consider are; the standard Merton(1974) model, the Black & Cox (1976) model with flat barrier, the Longstaff and Schwartz(1995)model with stochastic interest rates, the Collin- Dufresne and Goldstein (2001) model with stationary leverage, and the double exponential jump diffusion model used in Huang & Huang(2003). Our study provides consistent econometric estimation of the pricing model parameters and specification tests based on the joint behavior of time-series asset dynamics and cross-sectional pricing errors. Our empirical tests reject strongly the standard Merton (1974) model, the Black and Cox (1976) with flat barrier model, the Longstaff, Schwartz (1995) model with stochastic interest rates. The double exponential jump-diffusion barrier model used in (Huang and Huang, 2003) improves significantly over the five models. The best model is the stationary leverage model of Collin-Dufresne and Goldstein (2001), which we cannot reject at 0.5 level of significance in our sample firm. However, our empirical results document the inability of the existing structural models to capture the dynamic behavior of CDS spreads and equity volatility, especially for high investment grade names derivatives. These points to a potential role of time-varying asset volatility, a feature that is missing in the standard structural models.

Keywords: *Structural Credit Risk Model, Stochastic Volatility, Credit Default Swap Spreads, Consistent Specification Analysis; Pricing Error Diagnostics.*

INTRODUCTION

Credit derivatives markets have been growing exponentially over the past several years. According to the most recent biennial survey by the British Bankers' Association, the global credit derivatives market is expected to exceed $8 trillion in 2006. Credit default swaps (CDS) are currently the most popular credit derivatives instrument and account for about half of the credit derivatives market. Under a CDS contract the protection seller promises to buy the reference bond at its par value when a pre-defined default event occurs. In return, the protection buyer makes periodic payments to the seller until the maturity date of the contract or until a credit event occurs. This periodic payment, usually expressed as a percentage of the notional value underlying a CDS contract, is called the CDS spread. Compared with corporate bond spreads, CDS spreads are a relatively pure pricing of default risk of the underlying entity, abstracting from numerous bond characteristics, such as seniority, coupon rates, embedded options, and guarantees. As a result, there is a growing literature on testing credit risk models using the information from the CDS market. A widely used approach to credit risk modeling in practice is the so-called structural method, originated from Black and Scholes (1973) and Merton (1974). Whereas there have been many empirical studies of structural models, especially recently, based on corporate bond data, the empirical testing of these models using CDS spreads is quite limited. Such a testing is desirable especially given the recent empirical evidence based on the corporate bond market that existing structural models have difficulty either fitting corporate bond spreads (e.g., Jones, Mason, and Rosenfeld (1984), Lyden and Saraniti (2000), Delianedis and Geske (2001), Eom, Helwege, and Huang (2004), Arora, Bohn, and Zhu (2005) and Ericsson and Reneby (2005)) or explaining both spreads and default frequencies simultaneously (the so-called credit spread puzzle documented in Huang and Huang (2003)). If CDS spreads are considered to be a purer measure of credit risk than corporate bond spreads, then the existing structural models (purely default risk based) may perform better in capturing the behavior of CDS spreads than they do for corporate bond spreads. In this article we test five representative structural credit risk models using a sample of 93 single name CDS contracts during the period January 2002 - December 2004. The models we consider are the standard Merton (1974) model, the Black and Cox (1976) model with a flat barrier, the Longstaff and Schwartz (1995) model with stochastic interest rates, the Collin-Dufresne and Goldstein (2001) model with a stationary leverage, and the double exponential jump diffusion model used in Huang and Huang (2003).

More specifically, we formulate a specification test based on the pricing solutions of CDS spreads and equity volatility implied by a particular structural model. By assuming that both equity and credit markets are efficient and that the underlying structural model is correct, we obtain the identifying moment restrictions on the model parameters, such as asset volatility, the default barrier, and the speed of mean-reverting leverage. Such a GMM estimator with an ensuring J -test is a consistent econometric method, for parameter estimation and specification analysis of the structural credit risk models. One advantage of such a test is that it provides us with a precise inference on whether a particular structural model is rejected or not in the data, unlike the existing studies based on calibration, rolling estimation or regression analysis. Furthermore, unlike the existing studies that focus on 5-year CDS contracts, we use the entire term structure of CDS spreads. Such a method provides us a tighter identification of structural model parameters and minimizes the effect of measurement error from using bond characteristics, and thus attributes the test results mostly to the specification error. More importantly, by focusing on the equity volatility measured with high frequency data, instead of low frequency daily data, our approach speaks directly to the recent finding that volatility dynamics has a strong potential in better explaining the credit spreads.

Our empirical tests reject strongly the following three standard models: the Merton (1974) model, the Black and Cox (1976) model, the Longstaff and Schwartz (1995) model. However, the double exponential jump-diffusion barrier model outperforms significantly these three models. The stationary leverage model of Collin-Dufresne and Goldstein (2001) is the best performing one among the five models examined in our analysis and more specifically, is not rejected by the GMM test for more than half of the 93 companies in our sample. In addition, the test results allow us to gain a better understanding of the structural models, which otherwise does not obtain easily from *ad hoc* calibrations or rolling estimation analysis. For example, when allowing the default barrier to be different from the total liabilities, we discover a negative relationship between the observed debt/asset ratio and the implied default boundary trigger. Moreover, when a dynamic leverage or a jump component is allowed for, the overall fitting of average CDS term structure is improved with a much smaller pricing error. Further more, for the best performing dynamic leverage model, the individual firms sensitivity to interest rate or varies dramatically from significant positive for investment grade names to significant negative for speculative grade names, suggesting a great deal of heterogeneity in each firm's exposure to systematic risk.

Finally, our empirical analysis sheds some light on how to improve the existing structural models in order to fit better CDS prices. One implication from our results is that a term structure model more flexible than the one-factor Vasicek (1977) model – used in Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001) – may reduce the pricing error. Also judging from several pricing error diagnostics, jump augmentation seems to im- prove the investment grade names, while dynamic leverage seems to improve the speculative grade names. We also find that for the junk rated names, the observed spot leverage is very close to the long-run mean of the risk-neutral leverage implied by the Collin-Dufresne and Goldstein (2001) model; while for investment grades the spot leverage is much lower than the risk-neural leverages. This mirrors the recently documented low leverage puzzle for high rat- ing firms (Strebulaev and Yang, 2006; Chen and Zhao, 2006). Our analysis also documents the inability of the standard structural models in fitting time-series of both CDS spreads and equity volatility. Given that equity volatility in structural models is time-varying, this result provides a direct evidence that a structural model with stochastic asset volatility may improve the model performance (Huang and Huang, 2003; Huang, 2005; Zhang, Zhou, and Zhu, 2006).

Problem Statement/Justification

- *Inability of structural credit risk model to capture the dynamic behavior of CDS spreads and equity volatility, especially for high investment grade names derivatives*
- *we conduct specification analysis of structural credit risk models with stochastic volatility using term structure of Credit Default Swap (CDS) spreads and equity volatility form high-frequency return data*
- *We also test five representatives of structural credit risk models using samples of 93 single name CDS contracts from January 2010 -2022*

A Review of Structural Credit Risk Models

In the literature, the evaluation of structural credit risk models is generally based on comparing their pricing error on corporate bonds, although the models are typically not consistently estimated but rather judged based on *ad hoc* calibration or rolling sample extractions. Here we connect with the existing literature by looking at the pricing errors of researcher models, after the parameters are consistently estimated and model specification tests are conducted. If our approach is valid, then the specification test result should be consistent with the pricing errors evaluations. To be more specific, for each month and each maturity, we use the estimated structural parameters and pricing solutions to calculate the model implied CDS spreads and equity volatility. Then we compute the simple difference, absolute difference, and percentage difference between the models implied and observed ones. Finally the mean of the pooled pricing errors is reported for each name.

We consider five representative structural models in our empirical analysis. Specifically, they include the Merton (1974) model, the Black and Cox (1976) model, the Longstaff and Schwartz (1995) model, the Collin-Dufresne and Goldstein (2001) model, and the double exponential jump diffusion model considered in Huang and Huang (2003). The Black and Cox model with a flat barrier examined here can be also considered to be a special case of either the exogenous-default version of Leland and Toft (1996) or the one-factor version of Longstaff and Schwartz (1995). Except for the Merton model, all other ones are barrier-type models. Among the five models, Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001) are two-factor models, and the remaining three are one-factor models. For completeness, below we briefly review the five structural models to be tested in our empirical study. Although these five models differ in certain economic assumptions, they can be embedded in the same underlying structure that includes specifications of the underlying firm's asset process.

Campbell and Taksler (2003) find that idiosyncratic equity volatility can explain a significant part of corporate bond yield spreads cross-sectionally. Huang and Huang (2003) conjecture that a structural credit risk model with stochastic asset volatility may solve the credit spread puzzle. Huang (2005) considers an affine class of structural models with both stochastic asset volatility and L´evy jumps. Based on regression analysis, Zhang, Zhou, and Zhu (2006) provide empirical evidence that a stochastic asset volatility model may improve the model performance.

Methodology

Evaluation Methodology

We consider five representative structural models in our empirical analysis. Specifically, they include the Merton (1974) model, the Black and Cox (1976) model, the Longstaff and Schwartz (1995) model, the Collin-Dufresne and Goldstein (2001) model, and the double exponential jump diffusion model considered in Huang and Huang (2003) The Black and Cox model with a flat barrier examined here can be also considered to be a special case of either the exogenous-default version of Leland and Toft (1996) or the one-factor version of Longstaff and Schwartz (1995). Except for the Merton model, all other ones are barrier-type models. Among the five models, Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001) are two-factor models, and the remaining three are one-factor models. For completeness, below we briefly review the five structural models to be tested in our empirical study.

Although these five models differ in certain economic assumptions, they can be embedded in the same underlying structure that includes specifications of the underlying firm's asset process, the default boundary, and the recovery rate etc. Let V be the firm's asset process, K the default boundary, and r the default-free interest rate process. Assume that, under a risk-neutral measure,

$$d \ln K_t = \kappa_\ell [-\nu - \varphi(r_t - \theta_\ell) - \ln(K_t/V_t)]\, dt \text{ ---------------------(1)}$$
$$dr_t = (\alpha - \beta r_t)\, dt + \sigma_r dZ_t \text{ --------------------------------- (2)}$$

where δ, σ_v, κ_ℓ, θ_ℓ, ν, φ, α, β, and σ_r are constants, and W^Q and Z^Q_t are both one- dimensional standard Brownian motion under the risk-neutral measure and are assumed to have a constant correlation coefficient of ρ. In Eq. (1), the process N^Q is a Poisson process with a constant intensity $\lambda^Q > 0$, the Z^Q's are i.i.d. random variables, and $Y^Q \equiv \ln(Z^Q)$

I has a double-exponential distribution with a density given by

$$Q\,(y) = pu\ \eta u\ e \quad u \quad 1_{\{y\geq 0\}} + pd\ \eta d\ e\ d \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

In equation (3), parameters $\eta^Q, \eta^Q > 0$ and $p^Q, p^Q \geq 0$ are all constants, with $p^Q + p^Q = 1$.

All five models considered in this analysis are special cases of the general specification in Eqs. (1) - (3). For instance, if the jump intensity is zero, then the asset process is a geometric Brownian motion. This specification is used in the four diffusion models, namely, the models of Merton, Black and Cox (BC), Longstaff and Schwartz (LS), and Collin-Dufresne and Goldstein (CDG). Regarding the specification of the default boundary K, it is a point at the bond maturity in the Merton model. If κ_ℓ is set to be zero, then the default boundary is flat (a continuous barrier), an assumption made in Black and Cox (BC), Longstaff and Schwartz (LS), and the jump diffusion (HH) models. The mean-reverting specification in (2) is used in the Collin-Dufresne and Goldstein (CDG) model. The Vasicek model in (3) is used to describe the dynamics of the risk-free rate in the two-factor models of Longstaff and Schwartz (LS) and Collin-Dufresne and Goldstein (CDG) models. If both β and σ_r are zero, then the interest rate is constant, an assumption made in the three one-factor models. For simplicity and comparison with other studies, we assume a constant recovery rate.

Under each of the five structural models, we can calculate the corresponding risk-neutral default probability and then the CDS spread. Let $Q(t, \tau)$ denote the unconditional de- fault probability over $(t, t + \tau]$ under the risk-neutral measure (or the forward measure with stochastic

$$(1 - R)\ t{+}\tau\ e^{-rs} Q'(t, s)ds$$

$$\frac{r(1 - R)G(t, \tau),}{1 - e{-}r\tau\,[1 - Q(t, \tau)] - G(t,} \qquad (4)$$

where R is the recovery, r is the interest rate, and

$$G(t, \tau) = t{+}\tau\ e^{-rs} Q'(t, s)ds \qquad (5$$

With these identifying restrictions, we can build an internally consistent GMM estimator (Hansen, 1982), which minimizes the fitted errors of credit spreads and equity volatility, with an appropriate weighting matrix determined by the pricing model and data sample. Along with consistent parameter estimation, we obtain an omnibus specification test, to rank order various credit risk models and to judge their pricing performance in a systematic framework. In addition, we also use the term structure and time series of CDS spreads to evaluate the economic pricing errors, which should by-and-large confirm our GMM specification test results. A structural model would be rejected by the GMM criterion function test, if the pricing errors are relatively large and exhibit systematic variations, assuming that the equity and credit markets are efficient. The implementation of our estimation strategy has several advantages. First, we use high frequency equity returns to construct a more accurate estimate of the equity volatility, there- fore minimizing the measurement error imputed into the asset volatility estimate (given any structural model for the underlying asset process), while leaving the main suspect to possible model misspecification which we really care about. Second, we use the CDS spreads as a relative purer measure of the credit risk, therefore sanitizes our approach from the specific pricing error problem associated with bond market iniquity or other non-default charac- teristics (Longstaff, Mithal, and Neis, 2005). In addition, we use the term structure and time series of CDS spreads in both estimation and pricing exercise, while holding constant the model specification and parameter values, thus avoiding the rolling sample extraction approach that is inconsistent with economic assumption underlying the structural models. More importantly, by bringing in the consistency between observed equity and model im- plied equity, our approach has the potential to speak directly to the recent finding that time-varying equity volatility has a strong nonlinear forecasting power for credit With these identifying restrictions, we can build an internally consistent GMM estimator (Hansen, 1982), which minimizes the fitted errors of credit spreads and equity volatility, with an appropriate weighting matrix determined by the pricing model and data sample. Along with consistent parameter estimation, we obtain an omnibus specification test, to rank order various credit risk models and to judge their pricing performance in a systematic framework. In addition, we also use the term structure and time series of CDS spreads to evaluate the economic pricing errors, which should by-and-large confirm our GMM specification test results. A structural model would be rejected by the GMM criterion function test, if the pricing errors are relatively large and exhibit systematic variations, assuming that the equity and credit markets are efficient. The implementation of our estimation strategy has several advantages. First, we use high frequency equity returns to construct a more accurate estimate of the equity volatility, there- fore minimizing the measurement error imputed into the asset volatility estimate (given any structural model for the underlying asset process), while leaving the main suspect to possible model misspecification which we really care about. Second, we use the CDS spreads as a relative purer measure of the credit risk, therefore sanitizes our approach from the specific pricing error problem associated with bond market iniquity or other non-default charac- teristics (Longstaff, Mithal, and Neis, 2005). In addition, we use the term structure and time series of CDS spreads in both estimation and pricing exercise, while holding constant the model specification and parameter values, thus avoiding the rolling sample extraction approach that is inconsistent with economic assumption underlying the structural models. More importantly, by bringing in the consistency between observed equity and model im- plied equity, our approach has the potential to speak directly to the recent finding that time-varying equity volatility has a strong nonlinear forecasting power for credit

$$1 - R) \ e^{-rs}Q'(t, s)ds$$

$$\text{cds}(t, \tau) = \ e{-}rs[1 - Q(t, s)]ds \quad \text{---} \quad (6)$$

where r is the risk free rate, R is the recovery rate, $Q(t, s)$ is the model-dependent risk-neutral default probability at time t for period s, and $Q'(t, s)$ is the risk-neutral default intensity pointed out by Merton (1974), the delta function relating the equity volatility and asset volatility is also model-dependent the equity volatility of the continuous diffusion component satisfy Eq. (10). With observed CDS spread cds(t, τ) and equity volatility $\sigma_E(t)$, we can specify the following overidentifying restrictions
where θ is the structural parameter vector for various credit risk model under consideration, e.g., asset volatility, default barrier, asset jump intensity, or dynamic leverage coefficient, etc.. The term structure of CDS spread is represented by four maturities τ = 1, 3, 5, and 10 years.

Under the null hypothesis that the model is correctly specified, we have

$$E[f(\theta, t)] = 0. \qquad (7)$$

Note that both the CDS spread and the equity volatility are allowed to be observed with measurement errors. However, under the appropriately defined GMM metric, these pricing errors must be "small"; or the model specification will be rejected. The corresponding GMM estimator is given by

$$\hat{\theta} = \arg\min g(\theta, T)' W(T) g(\theta, T), \qquad (8)$$

where $g(\theta, T)$ refers to the sample mean of the moment conditions, $g(\theta, T) \equiv 1/T \quad T f(\theta, t)$, and $W(T)$ denotes the asymptotic covariance matrix of $g(\theta, T)$ (Hansen, 1982). With mild regularity conditions, the estimator of structural parameter θ is $\sqrt{T}$ -consistent and asymp- totically normally distributed, under the null hypothesis. Moreover, the minimized value of the objective function multiplied by the sample size

$$J = T \min g(\theta, T)' W(T) g(\theta, T) \qquad (9)$$

Result(Expected outputs/:

Rating	Firms	% of Sample	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout (%)	Recovery Rate (%)
AAA	1	1.08%	36.36	63.71	2.22	40.88
AA	6	6.45%	31.50	20.92	1.53	40.92
A	25	26.88%	32.51	38.15	2.02	40.57
BBB	45	48.39%	35.54	51.84	2.26	40.73
BB	11	11.83%	47.19	57.76	2.15	39.51
B	4	4.30%	83.23	72.61	2.28	38.23
CCC	1	1.08%	81.94	93.93	2.89	26.57
Overa Ll	93	100.00%	38.40	48.34	2.14	40.30

Data Description

Credit Default Swap Spreads

We choose to use the credit default swap (CDS) premium as a direct measure of credit spreads. CDS is the most popular instrument in the rapidly growing credit derivatives mar- kets. Compared with corporate bond spreads, which were widely used in previous studies in testing structural models, CDS spreads have two important advantages. First, a CDS spread is a relatively pure pricing of default risk of the underlying entity, and the contract is typically traded on standardized terms. By contrast, bond spreads are more likely to be affected by differences in contractual arrangements, such as seniority, coupon rates, embedded options, and guarantees.[3] Second, as shown by Blanco, Brennan, and March (2005) and Zhu (2006), while CDS and bond spreads are quite in line with each other in the long run, in the short run CDS spreads tend to respond more quickly to changes in credit conditions. This means that CDS market may be more efficient than bond market, therefore more appropriate for the specification tests of structural models.

Our CDS data are provided by Markit, a comprehensive data source that assembles a network of industry-leading partners who contribute information across several thousand credits on a daily basis. Based on the contributed quotes Markit creates the daily composite quote for each CDS contract; which must past the stale data test, flat curve test, and outlying data test. Together with the pricing information, the dataset also reports average recovery rates used by data contributors in pricing each CDS contract. In addition, an average of Moody's and S&P ratings is also included. In this paper we include all CDS quotes written on US entities (sovereign entities excluded) and denominated in US dollars. We eliminate the subordinated class of contracts because of their small relevance in the database and unappealing implication in credit risk pricing. We focus on CDS contracts with modified restructuring (MR) clauses, as they are the most popularly traded in the US market. We require that the CDS time series has at least 36 consecutive monthly observations to be included in the final sample. Another filter is that CDS data have to match equity price (CRSP), equity volatility (TAQ) and accounting variables (COMPUSTAT). We also exclude financial and utility sectors, following previous empirical studies on structural models. After applying these filters, we are left with 93 entities in our study. Our sample period covers January 2002 to December 2004, with maturities of 1, 2, 3, 5, 7, and 10 years.[4] For each entity, we create the monthly CDS spread by selecting the latest composite quote in each month, and, similarly, the monthly recovery rates linked to CDS spreads.

Equity Volatility from High Frequency Data

By the theory of quadratic variation, it is possible to construct increasingly accurate measure for the *model-free* realized volatility or average volatility, during a fixed time interval, say a day or a month, by summing increasingly finer sampled squared high-frequency returns (Andersen, Bollerslev, Diebold, and Labys, 2001b; Barndorff-Nielsen and Shephard, 2002;

Capital Structure and Asset Payout

Assets and liabilities are key variables in evaluating structural models of credit risk. The accounting information is obtained from Compustat on a quarterly basis and assigned to each month with the quarter. We calculate the firm asset as the sum of total liability plus market equity, where the market equity is obtained from the monthly CRSP data on shares outstanding and equity prices. Leverage ratio is estimated by the ratio of total liability to the firm asset. The asset payout ratio is proxied by the weighted average of the interest expense and dividend payout. Both ratios are reported as annualized percentages.

Risk-Free Interest Rates

To proxy the risk-free interest rates used as the benchmark in the calculation of CDS spreads, we use the 3-month LIBOR and the interest rate swaps with maturities of 1, 2, 3, 5, 7, and 10 years.

Empirical Results

In this section we summarize our empirical findings on testing the structural credit risk mod- els, based on the GMM estimator defined in Section 3 with the term structure of CDS spreads and equity volatility. We also provide some diagnostics on various model specifications based on the pricing errors, and discuss some implications for future research.

Conclusion

This research provides a consistent econometric specification test of five structural credit risk models using information from both the credit default swap (CDS) market and equity mar- ket. In particular, we consider the standard Merton (1974) model, the Black and Cox (1976) barrier model, the Longstaff and Schwartz (1995) model with stochastic interest rates, the stationary leverage model of Collin-Dufresne and Goldstein (2001), and the double expo- nential jump-diffusion barrier model studied in Huang and Huang (2003). We examine the performance of each model in capturing the behavior of CDS spreads and equity volatility both cross-sectionally and time series wise. Existing empirical studies of structural models mainly based on corporate bond spreads and equity volatility from low frequency daily data. To our best knowledge, this study is the first direct econometric estimation and specification test of structural models using data on the term structure of CDS and equity volatility estimated with high frequency intraday data. This allows us to minimize the effects of measurement error and pricing error, and thus attribute the test results mostly to the specification error We find that the Merton (1974), Black and Cox (1976), and the Longstaff and Schwartz (1995) models are strongly rejected by our specification test. The jump diffusion model considered in Huang and Huang (2003) improves the performance significantly for the top investment grade names but helps the fit mainly in the short end of the CDS term structure and not much in the long end. Still, the model is rejected for more than half of our sample firms. The best of the five models is the Collin-Dufresne and Goldstein model, that cannot be rejected in more than half of our sample firms. Nonetheless, we show that these structural models still have difficulty predicting credit spreads accurately even when CDS spreads (a purer measure of credit risk than bond spreads) are used in the analysis. Finally, we document that the five structural models cannot capture the time-series behavior of both CDS spreads and equity volatility. Given that equity volatility in structural models is time-varying, this finding provides a direct evidence that a structural model with stochastic asset volatility (see Huang and Huang, 2003; Huang, 2005; Zhang, Zhou, and Zhu, 2006) may significantly improve the model performance, especially for the investment grade names Longstaff, Mithal, and Neis (2005) find that a large proportion of bond spreads are deter- mined by liquidity factors, which do not necessarily reflect the default risk of the underlying asset. Additional maturities of 0.5, 15, 20, and 30 years are also available for the CDS data set. Due to the liquidity concern and missing value, we choose to focus on CDS with maturity between 1 and 10 years and Huang, 2004), therefore a more accurate measure of equity volatility from high-frequency data is critical in correctly estimating the asset return volatility — the driving force behind behind the firm default risk which converges to the integrated or average variance during period *t*. For the double- exponential jump-diffusion model, the continuous component of equity volatility (squared) can be estimated with the so-called “bi-power variation.

Summary Statistics

In this paper, we focus on the senior unsecured CDS contracts on U.S. corporations and denominated in U.S. dollars. Subordinated class of contracts are not considered here for their small representations in the fast growing CDS market and their complicated implications in credit risk pricing. We use only the modified restructuring (MR) clauses, as they are the most popularly traded in the U.S. market. After matching with the high frequency equity volatility and firm accounting information, excluding financial and utility firms, we are left with 93 entities spanning from January 2002 to December 2004. Table 1 provides summary statistics on CDS spreads and firm characteristics across both rating categories and sectors. As can be seen from panel A of Table 1, our sample is con- centrated in the single-A and triple-B categories, which account for 75 percent of the total sample, reflecting the fact that contracts on investment-grade names dominate the CDS market. In terms of the average over both the time-series and cross-section in our sample, the 5-year CDS spread is 144 basis points, equity volatility is 38.40 percent (annualized), the leverage ratio 48.34 percent, asset payout ratio 2.14 percent, and the quoted recovery rate 40.30 percent. As expected, the CDS spread, equity volatility, and the leverage ratio all increase as rating deteriorates. However, the recovery rate essentially decreases as rating deteriorates but has low variations Figure 1 plots both the term structure (from 1 year to 10 years) and time evolution (over the period from January 2002 to December 2004) of the average CDS spreads. As can be seen from the figure, the average spreads show large variations and have a peak around late 2002. Figure 2 plots both the 5-year CDS spreads and equity volatility by ratings over the entire sample period. The 5-year CDS spreads clearly have a peak in late 2002 across all three rating groups although the high-yield group has another spike in late 2004. On the other hand, equity volatility is much higher in 2002 than the later part of the sample period and, in particular, has two huge spikes in 2002.

GMM Specification Test

Our econometric method is based on the model implied pricing relationship for CDS spread and equity volatility. There is clear evidence that equity volatility and credit spread are intimately related (Campbell and Taksler, 2003), and the linkage appears to be nonlinear in nature (Zhang, Zhou, and Zhu, 2006). A casual inspection of Figure 2 indicates that CDS spreads and equity volatilities appear to move together sometime during market turmoils but are only loosely related during quiet periods. A structural model with richer time-varying feature in the underlying asset may be called for to account for the observed nonlinear relationship between equity volatility and credit spread.

The GMM specification test results from each of five structural credit risk models are given in Table 2. In particular, we report the percentage of firms where each of the five models is *not* rejected, for the whole sample as well as across both ratings and sectors. As can be seen from the table, none of the five models have a rejection rate of 100%. In order to estimate the stochastic interest rate model of Longstaff and Schwartz (1995) and the dynamic leverage model of Collin-Dufresne and Goldstein (2001), we need to first estimate the default-free term structure model of Vasicek (1977) in Eq. (3). Parameter estimates are obtained monthly based on cross- sectional data, and not reported here for brevity. The cross-sectional pricing errors that range from 12 to 112 basis points during the sample period. The existing empirical studies of the standard structural models based on corporate bond spreads have largely rejected these models as well. Our results indicates that the standard structural models are still missing something even when CDS spreads, presumably a cleaner measure of credit risk than corporate bonds spreads, are used in the empirical analysis.

Nonetheless, our empirical results provide new evidence on the relative performance of the five structural models and potential guidance on how to extend the existing models. For instance, notice that the GMM test statistics for the Merton (1974) specification are significantly higher than those for the other four extended models. (Some of the models are not nested so the J -test statistics are not always directly comparable.) Whereas it is known that the Merton model underperforms the richer models, our results are the first in the literature based on a consistent econometric test that takes into account the dynamic behavior of both CDS spreads and equity volatility.

Rating	Firms	% of Sample	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout (%)	Recovery Rate (%)
AAA	1	1.08%	36.36	63.71	2.22	40.88
AA	6	6.45%	31.50	20.92	1.53	40.92
A	25	26.88%	32.51	38.15	2.02	40.57
BBB	45	48.39%	35.54	51.84	2.26	40.73
BB	11	11.83%	47.19	57.76	2.15	39.51
B	4	4.30%	83.23	72.61	2.28	38.23
CCC	1	1.08%	81.94	93.93	2.89	26.57
Overall	93	100.00%	38.40	48.34	2.14	40.30
	Maturity of CDS					
Rating	1-year	2-year	3-year	5-year	7-year	10-year
	CDS Spreads Mean (%)					
AAA	0.23	0.28	0.32	0.43	0.45	0.49
AA	0.12	0.13	0.15	0.20	0.23	0.28
A	0.25	0.29	0.32	0.39	0.43	0.49
BBB	0.74	0.79	0.86	0.94	0.98	1.05
BB	2.62	2.74	2.84	2.90	2.92	2.92
B	7.52	7.20	7.51	7.25	7.01	6.79
CCC	25.26	22.99	20.91	18.81	18.03	17.31
Overall	1.34	1.36	1.40	1.44	1.45	1.49
	CDS Spreads Std. Dev. (%)					
AAA	0.17	0.19	0.21	0.25	0.23	0.24

Table 1: Summary Statistics on CDS Spreads and the Underlying Names

Panel B: By Industry

Sector	Firms	% of Sample	Equity Volatility (%)	Leverage Ratio (%)	Asset Payout (%)	Recovery Rate (%)
Communications	6	6.45%	48.72	42.93	1.99	40.14
Consumer Cyclical	32	34.41%	38.95	48.56	2.01	40.45
Consumer Staple	14	15.05%	33.77	41.68	2.24	40.87
Energy	8	8.60%	39.93	53.89	2.47	40.05
Industrial	18	19.35%	40.24	53.90	2.01	39.90
Materials	11	11.83%	32.85	49.34	2.73	41.35
Technology	4	4.30%	45.22	40.20	1.29	38.95
Overall	93	100.00%	38.68	48.39	2.14	40.39
Sector	Maturity of CDS					
	1-year	2-year	3-year	5-year	7-year	10-year
			CDS Spreads Mean (%)			
Communications	2.04	1.99	2.09	2.23	2.16	2.10
Consumer Cyclical	1.57	1.58	1.58	1.61	1.62	1.66
Consumer Staple	0.74	0.81	0.86	0.92	0.94	0.98
Energy	1.58	1.38	1.53	1.43	1.47	1.48
Industrial	1.29	1.38	1.41	1.46	1.48	1.53
Materials	0.92	0.96	1.03	1.10	1.14	1.20
Technology	1.38	1.43	1.48	1.48	1.51	1.52
Overall	1.34	1.36	1.40	1.44	1.45	1.49
			CDS Spreads Std. Dev. (%)			
Communications	4.82	4.13	4.58	4.74	4.33	3.80
Consumer Cyclical	6.19	5.25	4.65	4.06	3.85	3.65

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